

Chapter 4

Not Individuals, Nor Even Objects: On the Ontological Nature of Quantum Systems



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Abstract To which ontological category do quantum systems belong? Although we usually speak of particles, it is well known that these peculiar items defy several traditional metaphysical principles. In the present chapter these challenges will be discussed in the light of certain distinctions usually not taken into account in the debate about the ontological nature of quantum systems. On this basis, it will be argued that an ontology of properties without individuals, framed in the algebraic formalism of quantum mechanics, provides adequate answers to the ontological challenges raised by the theory.

4.1 Introduction

What kind of item is a quantum system? In the practice of physics it is common to speak of quantum particles, as if they were items of a similar nature to classical items, but obeying different laws of motion. However, as is well known, quantum systems have such peculiar features that they challenge certain ontological principles and categories as understood in traditional metaphysics. In general, these features are analyzed in the context of the so-called problem of indistinguishability, which is a consequence of the particular statistical behavior of quantum systems. But the fact that certain items are “indistinguishable” is not the only difficulty to be overcome in order to elucidate the ontological category of quantum systems.

On the other hand, the issue about the nature of quantum systems usually revolves around the category of individual: the question is whether this category can be applied to understand the peculiarities of quantum behavior. However, some metaphysical subtleties are overlooked in this discussion. What is the difference between the notions of object and of individual? What metaphysical principles are quantum systems supposed to fulfill? What ontological kinds of properties are

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involved in the quantum domain? These questions should be taken into account when the issue at stake is the elucidation of the ontological nature of quantum systems and, more generally, of the structure of the quantum realm.

The purpose of this chapter is twofold. On the one hand, different ontological distinctions that are not usually taken into account in the debate will be brought to the fore. In the light of these distinctions, the different ontological challenges that quantum mechanics poses to the ontological pictures presupposed by traditional metaphysics will be discussed. On the other hand, an ontology of properties, framed in the algebraic formalism of quantum mechanics, will be proposed. According to this view, quantum systems are not objects at all, but mere collections of properties that do not preserve their identity either synchronically or diachronically. The final aim of this work is to show how this proposal provides adequate answers to the ontological challenges raised by quantum mechanics to the categories coming from traditional metaphysics.

On the basis of this purpose, the chapter is organized as follows. In Sect. 4.2, certain metaphysical topics, usually ignored in the discussion on the interpretation of quantum mechanics, will be introduced and clarified. This clarification will allow us to discuss, in Sect. 4.3, the main quantum ontological challenges in more precise terms. Section 4.4 will be devoted to present the proposed ontology of properties, in the context of which quantum systems turn out to be non-objectual bundles of properties. This ontology will show its advantages to face the quantum ontological challenges in Sect. 4.5. Section 4.6 will delve into the physical nature of quantum systems conceived as non-objectual bundles. Finally, Sect. 4.7 will introduce some concluding remarks.

4.2 Some Metaphysical Preliminaries

4.2.1 *Ontological Categories*

In the discussions about the interpretation of quantum mechanics, it is commonly assumed that it introduces a deep break with respect to the classical view of reality. To recognize the extent of such a break, the first step is to recall what an ontological category is. A category is not a class defined by a concept, like “yellow” or “round”, which gathers certain objects together because they possess a certain property -or a cluster of properties-. A category is not a *taxon*, like “vegetal” or “mammal”, which classifies individuals into well-defined kinds. Categories are prior to any classification, since they are what endow reality with a certain structure. For this reason, they are conditions of possibility of any classification (Lewowicz, 2005).

Ludwig Wittgenstein (1921), in his *Tractatus Logico-Philosophicus*, introduces the distinction between *saying* and *showing*. A state of affairs external to language itself is “*sayable*”, and is depicted by a proposition. A proposition has a content that is fully intelligible to a person who is fluent in the language, even if she

does not know whether it is true or false. However, although propositions can depict the whole of reality, they cannot depict their own logical form, since this would require adopting a perspective outside of language itself. Thus, a proposition does not represent its logical form, but *shows* it in its own structure. In other words, the logical structure of language cannot be *said*; it can only be *shown*. As Wittgenstein states: “What can be shown, cannot be said.” (1921, Proposition 4.1212). These “unsayable things” are shown in the form of the propositions: they are there, in language, even though they cannot be said. In turn, the structure of language, shown by language itself, is also the structure of reality: “Propositions show the logical form of reality. They display it.” (Wittgenstein, 1921, Proposition 4.121). Therefore, the analysis of the logical structure of language allows us to understand the ontological structure of reality. A proposition as ‘the flower is pink’ says something, namely, it describes the fact that a certain flower is pink, but it cannot *say* that ‘flower’ is a noun representing an object and ‘pink’ is an adjective representing a property: the proposition can only *show*, through its structure, that it is speaking of a reality inhabited by objects and properties.

Once this Wittgensteinian distinction is taken into account, it becomes clear that categories are *said* neither with nouns nor with predicates nor with any other kind of word. *Categories are shown by language*: each language manifests, in its own structure, the categories that inform and organize the ontology to which it refers. For example, the structure of language will tell us whether the ontology is inhabited by objects, properties, and relations, whether ontological items can be categorized as one or multiple, whether events are temporally ordered, whether there are causal links between them. In turn, one task of metaphysics is to cut reality into categories. From Aristotle through Kant, many authors proposed categories to structure reality. Here only those categories that play a relevant role in the ontological interpretation of quantum mechanics will be considered, in particular, the categories of *object*, *property*, and *event*. I will use the term ‘item’ to refer to anything that exists, regardless of the category to which it belongs: objects, properties, and events are ontological items.

The ontological category of object is mirrored in language by the linguistic category of subject. As Ernst Tugendhat clearly explains: “There is a class of linguistic expressions which are used to stand for an object; and here we can only say: to stand for something. These are the expressions which can function as the sentence-subject in so-called singular predicative statements and which in logic have also been called singular terms” (Tugendhat, 1982: 23). This means that the category of object has its linguistic correlate in the so-called singular terms, which play the role of logical subjects of propositions and have singular references.

Since the category of object is closely related to the idea of subject of predication, it is always complemented by the ontological category of property: properties are attributed to objects and correspond to language predicates. In other words, if an object is a subject of predication at the logical level, at the ontological level it is the bearer of the properties attributed to it (see Rettler & Bailey, 2022). It is relevant to emphasize that monadic properties, usually called ‘properties’, and *n*-adic properties, usually called ‘relations’, both belong to the ontological category of

“property” and are distinguished by their arity. Or, in the other conceptual direction, a monadic property can be conceived as a particular case of relation, corresponding to $n = 1$.

On the basis of these distinctions, the most traditional metaphysical picture is that of an ontology of objects and properties. This picture is present, under different forms, in the Aristotelian dichotomy between primary substance and attributes and in Locke’s doctrine of the substance in general as the underlying substratum in which properties inhere, among many other cases in the history of philosophy. It is this kind of ontology that underlies Western ordinary languages and most systems of logic. In fact, in the propositions ‘Socrates is mortal’ and ‘water is liquid’, both of the form ‘S is P’, the linguistic distinction between the subject S and the predicate P expresses the ontological distinction between object and property. The same can be said in propositions with non-copulative verb, such as the case ‘The baby cries’: here too there is a subject, ‘the baby’, representing an object, and a predicate, ‘to be crying’, representing a property. Moreover, the predicate need not be monadic; in a relational proposition such as ‘Aristotle was the teacher of Alexander’, the names ‘Aristotle’ and ‘Alexander’ refer to objects, and the dyadic predicate ‘to be the teacher of’ denotes a relation.

The application of a monadic property to one object or of a n -adic property to n objects leads to a *fact*, which is logically expressed by a true proposition (Armstrong, 1993). Another ontological category is that of *event*, which is endowed with a strong temporal connotation. Some philosophers consider that the link between events and facts is close enough to justify the assimilation of the two categories (Wilson, 1974; Tegtmeier, 2000). However, facts are commonly conceived as having a certain temporal stability: they last for a certain period of time, in general not too short. By contrast, events are thought of as occurrences that are instantaneous or last for a very short time. On the basis of this temporal connotation, if events are assumed as a primitive ontological category, then temporal instants or intervals can be obtained as derived items, for instance, as maximal sets of simultaneous or partially simultaneous events (Russell, 1914; Whitehead, 1929), or time itself can be constructed as a linear ordering of events, induced by the binary relation “ x wholly precedes y ” between them (Thomason, 1989).

4.2.2 *Objects: Individuals Versus Stuff*

The traditional examples of objects are individual things, such as persons, tables, or flowers. However, if an object is understood strictly as a bearer of properties, the category of object is more general than that of individual: there are bearers of properties, such as water, soup, and jelly, which are not individuals but belong to a different subcategory. I will call it ‘*stuff*’ in order to avoid the philosophical and physical connotations of terms like ‘matter’ and ‘mass’.

An *individual* is an object that bears properties, but it needs additional features to be such. An individual is a *whole unity* in the sense that, as such an individual,

it is indivisible. This means that either it cannot be divided (it is “atomic”) or, if it can be divided, the results of the division are individuals or parts of a different kind from the original one. In turn, an individual is subject to the Kantian category of *quantity* (unity-plurality): individuals are either one (each one of them) or many, that is, a plurality. In the plural case, individuals form aggregates, in which they can be *counted*. As Henry Laycock says, the key to the character of this general category “evidently rests in the notions of unity and singularity—and thereby perhaps, more generally, in the concepts of number and countability.” (Laycock, 2010: 8).

In the Western philosophical tradition, the properties of an individual can be either (i) accidental, which are those that can change over time because the individual may or may not possess them, or (ii) essential, that is, those that the individual necessarily possesses and that in many cases allow the individual to be reidentified over time. In this sense, space-time properties always play a central role, either as essential properties of the individual or as the properties that confer individuality to the individual, in both cases under the assumption of impenetrability, which guarantees that two individuals cannot occupy the same place at the same time.

Most systems of logic include individual constants and individual variables, which represent items belonging to the category of individual. For instance, in first order logic, a proposition ‘ Pa ’ says that the property referred to by the predicate ‘ P ’ applies to the individual denoted by the individual constant ‘ a ’; in turn, in the expressions ‘ $\forall xPx$ ’ and ‘ $\exists xPx$ ’, the range of the individual variable x is a domain of individuals. The presence of individual constants and variables is not specific of traditional logic: the vast majority of systems of logic, even extensions of the traditional logic and deviant logics (see Haack, 1974, 1978), all include symbols to represent individuals, so that an ontology inhabited by individuals is presupposed. In turn, in traditional set theory, the elements of a set are also individuals: when we say that ‘ $a \in A$ ’, we mean that the individual denoted by ‘ a ’ belongs to the set represented by ‘ A ’, and this holds even in the case that the element denoted by ‘ a ’ is itself a set, since in this case the set behaves as an individual.

Even if it may be difficult to define what an individual is, it seems quite clear that the ontology we usually talk about includes individuals, precisely because the symbols used to denote them are ubiquitous in our ordinary and formal languages. Perhaps for this reason, the idea of an ontology of individuals and properties has been the dominant view in Western philosophical thought. It has also shaped physics since Modern times, from the corpuscular philosophy of Galileo and Boyle, up to present-day physics, with the standard model of fundamental particles. On the contrary, the answer to the question ‘What is stuff?’ is far from easy, as there is no strong tradition to help us. Laycock introduces the issue with an everyday example: “Removing a fly from a bowl of soup inevitably involves removing some soup as well; but it seems grammatically inappropriate to say that in such a case, there is another thing which is removed, alongside the fly. [...] the spatio-temporal isolation of any such soup will be arbitrary or adventitious [...] that, in essence, is why soup must be served in discrete bowls.” (Laycock, 2010: 15). The key difference between the two cases is that a fly is an individual, whereas soup is stuff.

As in the case of individuals, a *stuff* must have an identity principle, that is, a principle that distinguishes it from other stuffs of kinds of stuff. However, in this case such a principle has nothing to do with space and time: what distinguishes water from alcohol has no relation to spatio-temporal properties. Despite this, *portions of stuff* do exist in space and time: a portion of water can be located spatio-temporally, for example, now and here, in my glass. Unlike individuals, a portion of stuff can be further divided into portions of the same stuff, that is, it *can be divided* without losing its identity: if a piece of chalk is broken into smaller pieces, the resulting parts are also pieces of chalk. However, a stuff is not each of its portions: the meaning of ‘chalk’ cannot be established by pointing to one piece of chalk. But neither is a stuff the mere aggregate of its portions: the reference of the word ‘water’ is something beyond all the portions of water that exist in the universe. This means that a stuff embodies *unity and multiplicity* at the same time: it is one stuff, but it has multiple manifestations in its portions. Despite this, portions of stuff do not behave as individuals because, when they are put together in an aggregate, they *cannot be counted*: the aggregate of two portions of water is not “two waters” but “more water”. Moreover, whereas individuals preserve their identity in the aggregate, portions of stuff cannot be reidentified once they are put together: it cannot be said that “this” is one and “that” is the other of the original portions of, for instance, water or iron (see Lewowicz & Lombardi, 2013).

Analytic philosophy has faced the problem of understanding the category of stuff from a linguistic perspective through the discussion of the so-called “*problem of mass terms*”. In the words of Donald Davidson (1967): the problem is to understand the difference between countable nouns and mass nouns or non-countable nouns (see Pelletier, 1979). In fact, a mass term refers to something that cannot be counted. It is in this sense that mass terms are said to have the semantic property of referring cumulatively: “any sum of parts which are water is water” (Quine, 1960: 91). Like plural nouns, mass terms are semantically *non-singular*, a fact reflected in their non-acceptance of singular determiners: one can speak of ‘all water’, ‘some water’ and ‘more water’, but not of ‘a water’, ‘each water’ or ‘one water’. As a consequence, mass terms do not denote individual portions of stuff. On the other hand, they have in common with singular nouns the distinction of being semantically *non-plural*: whereas we can say ‘all oranges are sweet’, we can only say ‘all water contains impurities’. This means that the reference of mass terms is neither singular nor plural, since they designate neither one nor many individual things: “we should not expect a successful reduction to singular reference and singular predication, something that the application of traditional first-order logic would require [...] when we say that water surrounds our island [...] our discourse is not singular discourse (about an individual) and is not plural discourse (about some individuals); we have no single individual or any identified individuals that we refer to when we use ‘water’” (McKay, 2008: 310–311).

Summing up, when the category of object is in the spotlight, it is necessary to consider the distinction between individual and stuff, since the two subcategories lead to very different ontological pictures.

4.2.3 Objects: *Metaphysical Identity*

The question of identity is a traditional topic of metaphysics. A distinction is commonly made between *qualitative identity* and *numerical identity* (Noonan & Curtis, 2022). Qualitative identity is a relationship between two objects that have certain properties in common, and is a matter of degree. Numerical identity, on the other hand, is a relationship that an object has only with itself, and involves absolute qualitative identity. The present work is concerned exclusively with numerical identity, so the term ‘identity’ will always be used in the sense of numerical identity. It is in this context that different criteria of identity are proposed. A criterion of *synchronic identity* makes it possible to identify an object by distinguishing it from all others at a given instant of time. A criterion of *diachronic identity* makes it possible to reidentify an object over time. Commonly, identity criteria are discussed in the case of individuals; here we will consider them in the more general case of objects.

When the criterion of synchronic identity depends on an ontological category that transcends the properties of the object, a *transcendental identity* (transcendental individuality in terms of Post, 1963) is established. Duns Scotus’s *haecceitas*, a notion still present in contemporary metaphysics, is an example of this approach; substance in Locke’s sense is another typical example (Kaplan, 1975). By contrast, for the *bundle theory*, the criterion depends exclusively on the properties of the object (Armstrong, 1989), since the object is nothing but a bundle of properties. In this case, identity is defined as the equivalence relation (reflexive, symmetric and transitive) satisfying Leibniz’s law (conjunction between the principle of identity of indiscernibles and the principle of indiscernibility of identicals).

The principle of indiscernibility of identicals states that if x and y are identical (they name the same object), then exactly the same properties apply to them (this is a case of absolute qualitative identity). Assuming that the identity relation satisfies this condition is not controversial. However, this principle alone cannot act as a criterion of identity, since the identity relation appears in its antecedent. The converse of this principle, the *principle of identity of indiscernibles* (PII), states that if exactly the same properties apply to x and y , then they are identical. The PII could work as an identity criterion, since the identity relation appears in its consequent. However, in the specification of the minimal subset of properties that suffices to obtain the identity, a new problem arises. In fact, three versions of the PII are distinguished on the basis of the subset of properties considered relevant (French & Krause, 2006): (i) PII(1), if two objects have all their monadic and relational properties in common, then they are identical, (ii) PII(2), if two objects have all their monadic and relational properties in common, except spatio-temporal properties, then they are identical, and (iii) PII(3), if two objects have all their monadic properties in common, then they are identical. These three versions are ordered from lowest to highest logical strength. PII(2) and PII(3), being stronger, have counterexamples in the classical (non-quantum) domain; the traditional one is the case of Black’s spheres (Black, 1952): two exactly equal spheres in an empty

Newtonian space, sharing all their monadic and relational properties, except the spatio-temporal ones, and yet they are two objects. $\text{PII}(1)$, being the weakest, is the only version that is a candidate for metaphysical necessity.

Diachronic identity, although not as intensely discussed as synchronic identity, is not less relevant. What is it that makes an object the same at different times? Proponents of transcendental identity may claim that substance or haecceitas is what persists over time and thus reidentifies the object through change. Those who adopt the bundle theory are committed to selecting certain essential properties that do not change over time, while accidental properties may be replaced. In the case of the bundle conception of individuals, space-time properties are usually conceived as the essential properties that confer diachronic identity in terms of their continuity (see Gallois, 2016).

4.2.4 *Properties: Universals Versus Tropes*

Properties are items that are present in almost any ontological picture, since they are necessary for classification: objects that are numerically different can, nevertheless, be similar in terms of a certain feature or characteristic and, as a consequence, can be grouped in the same class. However, despite their ubiquity, the metaphysical nature of properties has been the subject of controversy since the origin of philosophical thought.

Since Plato, who used the term ‘εἶδος’ to designate them, properties have traditionally been conceived as *universals*, that is, items that can be shared by different objects, in contrast to particulars, which have a single existence. The peculiarity of universals is that they are “one-in-many”: a universal is one (e.g. redness), but it is instantiated in a multiplicity of cases (red in this case, red in that case). A universal is fully present in each of its instances, and the existence of a universal in one case is unrelated to its simultaneous existence in another case (see MacLeod & Rubenstein, 2006). The so-called “*problem of universals*” has permeated the history of philosophy from its beginnings, with questions about whether and how universals exist. There are three long-standing answers to these questions: *realism*, *nominalism*, and *conceptualism*. Whereas realists accept universals, *conceptualists* and *nominalists* refuse to accept their existence. Conceptualists explain the similarity between individuals by appealing to general concepts that exist only in minds. Nominalists, on the other hand, leave the relation of qualitative similarity as a brute and primitive fact. According to realists, by contrast, universals exist as mind-independent items. For *transcendent* realism, such as that proposed by Plato, they exist even though they are not instantiated, and they are thus “transcendent” or “*ante res*” (“before the things”). For *immanent* realism, such as that defended by Aristotle, universals are “immanent” or “*in rebus*” (“in things”), since they exist only if they are instantiated by objects. Whatever the differences between the two forms of realism, in both cases the *instances of a universal property* are many but absolutely indistinguishable: they are *only numerically different*.

Since a universal has multiple instantiations, it is necessary to elucidate the concept of *instantiation*. Instantiation is traditionally conceived as the relationship that a universal property maintains with the substratum to which it is “attached” (see Orilia & Paoletti, 2022). However, from a bundle-theory viewpoint, there are no substances, substrata, or individuals in which the properties inhere. Therefore, in this case, instantiation is the relationship that a property maintains with its multiple manifestations, called ‘instances’. For example, the property “red” has many instances in different situations, that is, when it appears in different bundles. In the physical realm, energy and momentum are properties that are instantiated in their multiple empirical manifestations.

In twentieth-century metaphysics, a new approach to properties entered the scene: properties as *tropes*, a stance that claims to occupy a middle position in between realism and nominalism with respect to universals. Tropes are *particular* properties, such as the particular shape, weight, and texture of an object (see Maurin, 2018). That two objects “share” a property (for example, a particular shade of redness) means that they each exemplify a redness-trope, where the two redness-tropes, though numerically distinct, nevertheless resemble each other exactly. Because tropes are particulars, trope theorists face the problem of providing a principle of individuation for tropes (see Schaffer, 2001). A natural answer is to individuate tropes by reference to the objects that instantiate them. However, this strategy does not work in the context of a bundle view: if objects are bundles of tropes, that principle of individuation becomes circular. A different approach appeals to space-time individuation, according to which two tropes are different when they are located at different space-time positions: the redness-trope now and here as different from the redness-trope then and there (see, e.g., Campbell, 1990). But this view needs to exclude space-time position as a property in the same sense as the rest of the trope-properties. For these and other reasons, many trope theorists have opted for a primitivist perspective, according to which the fact that two tropes are distinct is a primitive fact lacking any further metaphysical explanation (see, e.g., Keinänen & Hakkarinen, 2014).

4.2.5 *Properties: Determinables and Determinates*

Another distinction regarding properties, which is often not sufficiently taken into account, is the traditional difference between *determinables* and *determinates*, that is, properties that stand in a distinctive specification relation; let us call it ‘*determination*’ (see Wilson, 2022). For example, *color* is a determinable having *red*, *blue*, and other specific shades of color as determinates; *shape* is a determinable having *rectangular*, *oval*, and other specific shapes (including many irregular ones) as determinates; *mass* is a determinable having specific mass values as determinates. The determination relation differs from other specification relations. Unlike the genus-species and conjunct-conjunction relations, in which the more specific property can be understood as a conjunction of the less specific property and

some independent property or properties, a determinate cannot naturally be treated in conjunctive terms (whereas man can be conceived as the conjunction between animal and rational, *red* is not a conjunctive property having *color* and some other property or properties as conjuncts). And, unlike the disjunct-disjunction relation, in which disjuncts may be dissimilar and compatible (as in the case of ‘*red or round*’), the determinates of a determinable are both similar and incompatible (*red* and *blue* are similar in the sense that they are both colors, but nothing can be simultaneously and uniformly both *red* and *blue*).

It is extremely important to emphasize that the determinable-determinate relationship should not be confused with the universal-instance relation. The latter is the relation between a universal property and its many instances: for example, the universal property “color” has countless instances of colored items. The first is the relation between a property and other more specific properties that are cases of it: for example, the determinable “color” has “red”, “green”, “yellow”, etc. as its determinables. The distinction between the two kinds of relationship is clear in a classical ontology. In fact, the position Q and the momentum P of a particular classical object, say, this billiard ball, are instances of the universal properties “position” and “momentum”, respectively. In turn, the position $q_1 = “10 \text{ cm from the corner of the table}”$ and the momentum $p_1 = “20 \text{ gr cm/sec with respect to the table}”$ (in both cases, the billiard table is taken as the reference frame) are determinates of the determinables $Q = “position of the billiard ball”$ and $P = “momentum of the billiard ball”$, respectively. Furthermore, those determinates define the classical state s_1 of the billiard ball, conceived as a classical system, at a given time t_1 : $s_1 = (q_1, p_1)$.

The *principle of omnimode determination* is a principle that was generally accepted in Modern philosophy. For example, it already appears in the works of Wolff: “*Apparet hinc, individuum esse ens omnimode determinatum*” (“Hence it appears that an individual is a completely determined being”) (Wolff, 1728, p. 152). It can also be found in Bernoulli’s famous treatise on the calculus of probabilities: “*Sed quicquam in se et sua natura tale esse [viz. incertum et indeterminatum], non magis a nobis posse concipi, quam concipi potest, inde simul ab Auctore naturæ creatum esse et non creatum*” (“That anything is uncertain and indeterminate in itself and by its very nature is as inconceivable to us as it would be inconceivable for that thing both to have been created and not created by the Author of nature”) (Bernoulli, 1713, p. 227). It is also repeated several times by Kant in his lectures on metaphysics: “*Alles, was existirt, ist durchgängig determinirt*” (“Everything that exists is continuously determined”) (1902, AA 18:332, 5710; AA 18:346, 5759; see also LM XXVIII 554). The idea is that, in any object, all determinables are determinate: if the determinable “color” applies to an object, the object necessarily has some determinate color, say red, independently of its other determinate properties, and also independently of our knowledge about what that determinate color is. In other words, it is not possible for an object to have a determinable property that is not determinate: an object cannot be colored without being of some particular color, say, red, blue, white, etc.

4.3 The Ontological Challenges of Quantum Mechanics

The philosophical considerations introduced in the previous section are not a mere exercise in metaphysical thinking, but must be taken into account when specifying the structure of the quantum ontology. As will become clear in the present section, they are especially relevant for a clear understanding of the ontological challenges posed by quantum mechanics.

4.3.1 Contextuality

One of the first reactions to the probabilistic character of quantum theory was the attempt to interpret it as a statistical theory, in the style of classical statistical mechanics, so that the probabilities could be explained as frequencies in ensembles of systems with definite but “hidden” values of their observables. The *coup de grace* for such classical-style statistical interpretations was the Kochen–Specker theorem (Kochen & Specker, 1967), which proves the impossibility of ascribing precise values to *all* the observables of a quantum system simultaneously while preserving the functional relations between commuting observables. It follows that the selection of observables to which precise values can be attributed must be contextual, i.e. situation-dependent (e.g., dependent on the measurement context).

In the discussions on the interpretation of quantum mechanics, the distinction between determinable and determinate, which allows us to formulate the principle of omnimode determination, is almost never taken into account. This is surprising because quantum contextuality defies precisely that traditional and intuitive principle: while in all classical objects all determinables are determinate, in the quantum realm non-commuting observables correspond to determinables that are not simultaneously determinate. Furthermore, according to the Kochen–Specker theorem, a quantum system always have determinables (observables in the physical language) that are not determinate (that do not have precise values). It is interesting to note that the breach of the principle of omnimode determination is counterintuitive for any kind of object, not only for individuals, but also for stuff. In fact, one expects that not only in the case of a billiard ball but also in the case of soup, the determinable property, say, “color”, is determinate, say, “white”.

Different approaches have been proposed to accommodate quantum contextuality. One of them is based on the adaptation of the logic used in the quantum framework: starting from the fact that contextuality is related to the non-Boolean structure of elementary quantum propositions, a non-classical propositional logic can be formulated in terms of the non-distributive, orthocomplemented lattice of the theory (see, e.g., Jauch & Piron, 1969; Piron, 1976; Beltrametti & Cassinelli, 1981). From a more physical perspective, other authors have dealt with quantum contextuality by selecting a context, via an interpretive assumption (see, e.g., Bub & Clifton, 1996; Dieks, 2005), or via a physical process such as decoherence (see,

e.g., Zurek, 1982, 2003). However, the general problem of what a quantum system *is* and what structure the quantum ontology should have for contextuality to be natural has not been answered in a systematic way.

4.3.2 *Non-locality and Non-separability*

Unlike the classical world, the quantum domain admits surprising correlations between the properties of distant non-interacting systems, such as those of the famous Einstein-Podolsky-Rosen experiment. Taken at face value, EPR-correlations strongly suggest non-locality, that is, *non-local* influences between spatially distant systems, i.e., systems between which no light signal can travel. However, since this idea is incompatible with special relativity, the exact nature of those quantum correlations is a subject of ongoing controversy (see Berkovitz, 2016). For instance, according to collapse interpretations, EPR-correlations imply a certain action at a distance which, nevertheless, does not allow sending information at a superluminal velocity. In the case of Bohmian mechanics, it is the quantum field that possesses the necessary non-local features to induce EPR-correlations. From another perspective, those correlations are consequences of the holistic nature of quantum systems, understanding holism as the opposite of separability. *Separability* implies that, if a physical object is constructed by assembling its physical parts, then its physical properties are completely determined by the properties of the parts and their relationships. *Holism*, by contrast, is the characteristic of some physical objects that are not composed of physical parts, but are indivisible wholes; so, EPR-correlations are correlations between properties of a single holistic object (see Healey & Gomes, 2022).

From the viewpoint of the state of the composite system, correlations leading to non-locality and non-separability appear when the state is entangled. However, when discussing this issue *the relativity of entanglement* is rarely taken into account. As John Earman (2015) clearly stresses, a given state is entangled or not only in relation to the decomposition of the composite system into subsystems. In fact, a given state may be entangled with respect to a certain decomposition and non-entangled with respect to another. The typical case is that of the hydrogen atom, which can be decomposed into the proton-system and the electron-system, but also into the center of mass-system and the relative-system: the entanglement of the atom's state is relative to the chosen decomposition (see, e.g., Harshman, 2012). It has even been proven that, given a state vector $|\Psi\rangle$ in a finite-dimensional state space $\mathcal{H}$ with non-prime dimension $d = m \cdot n$, there always exists a partition, expressed by a tensor product structure $\mathcal{H} = \mathcal{H}^m \otimes \mathcal{H}^n$, with respect to which $|\Psi\rangle$ is factorizable and, therefore, non-entangled (Terra Cunha et al., 2007). And since “without further physical assumption, no partition has an ontologically superior status with respect to any other” (Zanardi, 2001: 4), there is no reason to privilege one claim about the entanglement of a quantum state over others. This relativity makes it difficult to conceive the correlations due to entanglement as consequence of non-local

influences between subsystems of a composite system, since the very idea of well-defined subsystems goes into crisis. By contrast, entanglement-induced correlations favor a holistic view of the composite system, according to which correlations are internal to a single indivisible item.

4.3.3 *Indistinguishability*

Most discussions about the ontological commitments of quantum mechanics focus on the challenge posed by the indistinguishability of so-called “identical particles” (particles of the same kind, that is, with the same state-independent properties) to the ontological category of individual. The usual story begins by counting how many distributions or complexions of two particles over two states are possible. The classical answer is given by the Maxwell–Boltzmann statistics, according to which there are four possible distributions of two individuals over two states. By contrast, in quantum statistics (Bose–Einstein and Fermi–Dirac), a permutation of the particles does not lead to a different complexion since particles are “indistinguishable”. Although the theory has formal resources to deal with quantum statistics, from a conceptual viewpoint the problem is to explain why a permutation of individual particles does not lead to a different complexion in the quantum case.

Indistinguishability is often considered to be a feature that leads to the violation of the weakest version of the principle of identity of indiscernibles and, consequently, that challenges the category of individual. Already in the 1960s, Heinz Post (1963) argued that elementary particles cannot be regarded as individuals, but must be viewed as “non-individuals”: this led to the so-called “Received View” about quantum indistinguishability (see French, 2019). However, in general the Received View gives no metaphysical characterization of those items beyond their non-individuality: they are only negatively characterized. An exception is the case of Paul Teller (1998), who proposes an account of quantum indistinguishability in terms of stuff: if quantum objects are stuff, permutation invariance follows naturally, because permuting two portions of the same stuff over two states does not give rise to a different complexion.

Several perspectives confront the Received View in order to restore the category of individual. For example, Bas van Fraassen (1985) recovers quantum statistics by renouncing the equiprobability of the different distributions of quantum particles in quantum states. Another position contrary to the Received View comes from Steven French (1989), who claims that states of indistinguishable particles that are not symmetric or antisymmetric are ontologically possible and only physically inaccessible: indistinguishability is a physical situation, not an ontological condition from which non-individuality can be inferred. Another alternative proposal to the Received View is based on the idea of weak discernibility (Saunders, 2003; Muller & Saunders, 2008): in the case of two fermions in a singlet state, the relation “having the opposite direction of each spin component with respect to...” that each fermion has with respect to the other is sufficient to establish numerical distinction between

the objects, even if they are indistinguishable with respect to their monadic and relational properties (for an analogous argument for bosons, see Muller & Saunders, 2008). French and Décio Krause (2006) have rejected this strategy by claiming that it entails circularity: in order to appeal to such relations, one has had to discriminate beforehand between the related objects; therefore, the numerical difference between the objects has been presupposed by the relation itself.

Although the problem of indistinguishability in quantum mechanics has generated extensive discussions, a common ground underlies all of them. In fact, despite the Received View and its critics differ with respect to conceiving quantum systems as individuals, they nevertheless agree with respect to subsuming them under the category of object.

4.4 Systems as Non-objectual Bundles of Properties

Hardly anyone denies the underdetermination of metaphysics by physics: quantum mechanics is compatible with distinct metaphysical “packages”. In fact, the quantum domain has been conceived as structured on the basis of very different fundamental ontological categories.

According to Bohmian Mechanics (see Dürr et al., 2013), the universe is a configuration of particles in precise positions relative to each another. Therefore, elementary quantum systems are *individual particles* with synchronic and diachronic individuality: they are discernible by their positions in the configuration, and they can be reidentified over time by the continuous trajectory traced by their motion. The difference with respect to the classical case is that the dynamics of quantum individuals is described by a law of motion known as “guiding equation”, which makes the evolution of each particle depend on the position of all the others, through the wave function. Proponents of Bohmian Mechanics believe that it provides the smallest deviation from the ontology of classical mechanics that is necessary to accommodate quantum phenomena (see, e.g., Esfeld, 2019).

The Ghirardi-Rimini-Weber (GRW) collapse theory (Ghirardi et al., 1986) states that, in order to solve the quantum measurement problem, the dynamical equation of the standard theory must be modified by adding stochastic and nonlinear terms. This new equation describes the spontaneous jumps undergone by the wave function in configuration space at random times. This theory has been ontologically interpreted in two different ways: *the matter-density ontology* (GRWm) and *the flash ontology* (GRWf). In GRWm, the wave function describes a continuous matter density field (see Ghirardi et al., 1995), which varies at different points of the three-dimensional physical space and changes in time. This means that there is a single fundamental object in the universe, a *stuff* that “fills” the entire space. The defined outcomes of measurements are the result of the spontaneous contraction of the matter density field at certain points or regions of space: the collapse of the wave function represents such a contraction of the matter density field. Discrete individual objects are mere appearances that arise in certain regions of the physical space where

the matter density is higher. According to GRWf, by contrast, each jump of the wave function in configuration space represents an *event* occurring at a point of physical space. These point-events are known as “flashes” (Tumulka, 2006). The time evolution of the wave function in configuration space represents the probability of the occurrence of future flashes, given an initial distribution of flashes. Therefore, physical space is not filled with objects, neither individuals nor stuff, but with a sparse distribution of discrete events. Objects are, from this view, nothing but clusters of a huge number of flashes.

These cases show that the literature on the ontology of quantum mechanics has appealed to different fundamental ontological categories in order to design the structure of the quantum domain: individuals and properties in Bohmian Mechanics, stuff in the matter-density ontology, events in the flash ontology. However, the possibility of an ontology in which properties are the only fundamental items has scarcely been considered. In fact, the idea of bundle of properties has appeared only a few times in the literature on quantum physics. It has been proposed for quantum field theory in its algebraic version by Meinard Kuhlmann (2010), and suggested by Cord Friebe (2014) in his objections to Gian Carlo Ghirardi’s criterion for entanglement of indistinguishable particles. In this scarcity lies the novelty of the proposal for an ontology of properties, which was originally presented by Olimpia Lombardi and Mario Castagnino (2008) in a paper on the Modal-Hamiltonian Interpretation of quantum mechanics, and was subsequently developed in later works (da Costa et al., 2013; da Costa & Lombardi, 2014; Lombardi & Dieks 2016; Fortin & Lombardi, 2022). This section will briefly introduce the proposal, highlighting its ontological significance and conceptual connotations.

4.4.1 Formalism and Ontology

At present it is clear that a formalism does not determine its interpretation: if a formal system has one interpretation, it may have an infinite number of interpretations. Nevertheless, this does not imply that formalisms are ontologically neutral: different formal systems, even if equivalent, may suggest different ontologies. A typical example is the theory of natural numbers, which can be formulated either on the basis of Peano’s axioms or in terms of Russell’s set-theoretic construction: although mathematically equivalent, the two formulations have different ontological connotations. From Peano’s perspective, natural numbers admit a realist, Platonist interpretation, according to which they exist as abstract entities. Russell’s formulation, by contrast, is more favorable to a nominalist interpretation, according to which reality is populated by individuals and classes, but not by natural numbers. An example from physics is the case of Hamiltonian and Lagrangian classical mechanics, which are also mathematically equivalent: they lead to the same predictions but evoke different ontological pictures. Hamiltonian mechanics gives a dynamical picture: it describes time processes that, starting from an initial state, all evolve in the same temporal direction under the rule of a

dynamical law. Lagrangian mechanics, by contrast, suggests a static picture, with a crystalized time, where there are two states at once, and the trajectory between them is given by a variational principle, in particular, the principle of least action, which selects one among all possible trajectories. Some prominent physicists have even endowed Lagrangian mechanics with a teleological meaning, regarding it as reintroducing final causes in physics (see discussion in Ben-Menahem, 2018, in particular, Planck quote in page 150).

Analogously to the above cases, different formalisms for standard quantum mechanics, although mathematically equivalent, design different ontological pictures. In the Hilbert space formalism, a quantum system is represented by a Hilbert space, whose vectors represent the system's states; observables are represented by operators acting on the Hilbert space. The mathematical priority of systems with their states over observables is easily reflected in an ontology of individuals, endowed with ontological priority over their properties. By contrast, in the *algebraic formalism*, a quantum system is represented by an algebra of observables, and states are functionals on that algebra. If this mathematical *priority of observables over states* is transferred to the ontological domain, the result is an ontology whose primary items are properties, and systems arise from the convergence of those properties.

In the algebraic framework, a quantum system is represented by a $*$ -algebra $\mathcal{A}$ of observables $A \in \mathcal{A}$, closed under products, linear combinations, and involution. A state of the system is represented by a normed and positive expectation-value functional $\omega : \mathcal{A} \rightarrow \mathbb{C}$ belonging to the dual algebra $\mathcal{A}'$. A state ω is pure when it cannot be written as a non-trivial convex combination $\omega = \lambda_1 \omega_1 + \lambda_2 \omega_2$, with $0 < \lambda_1, \lambda_2 < 1$, $\lambda_1 + \lambda_2 = 1$, and $\omega_1, \omega_2 \in \mathcal{A}'$; otherwise ω is mixed. The Gelfand-Naimark-Segal (GNS) construction (Gelfand & Naimark, 1943, Segal, 1947) proves that, if $\mathcal{A}$ is a C^* -algebra, then it can be mathematically represented by a set $\mathcal{O}$ of Hermitian operators O on a Hilbert space $\mathcal{H}$, and states can be mathematically represented by normed trace (density) operators ρ on $\mathcal{H}$. When the state ω , represented by the density operator ρ , is pure, then there is a vector $|\Psi\rangle \in \mathcal{H}$ such that $\rho = |\Psi\rangle\langle\Psi|$. For different $*$ -algebras, other representations of the algebra have been proved; for instance, a nuclear algebra can be represented by a rigged Hilbert space (see Iguri & Castagnino, 1999; for applications of rigged Hilbert spaces to quantum mechanics, see Bohm & Gadella, 1989). From now on, we will not distinguish between the abstract algebraic language and the language of the mathematical representation; then, we will say that a quantum system is represented by the algebra $\mathcal{O}$ of observables $O \in \mathcal{O}$, and that the system's states are expectation-value functionals $\rho(O) = \text{Tr}(O\rho) = \langle O \rangle_\rho \in \mathbb{R}$, for all $O \in \mathcal{O}$. Moreover, given two component systems represented by the algebras of observables $\mathcal{O}^1$ and $\mathcal{O}^2$, the composite system is represented by $\mathcal{O}^1 \vee \mathcal{O}^2$, that is, the minimal algebra generated by $\mathcal{O}^1$ and $\mathcal{O}^2$. In turn, we will also not distinguish between the physical language (e.g., observables, states) and the mathematical language (e.g., elements of an algebra, functionals on an algebra), under the assumption that the context clarifies the meaning of each term.

It is not necessary to delve into the technical details of the formalism to emphasize that the algebraic approach brings to the fore the difference between observables and states, whose corresponding spaces may not even be the same; this is precisely the case in nuclear algebras, represented by rigged Hilbert spaces. In the algebraic theoretical framework, observables are the basic elements of the theory; states are secondary elements, defined in terms of the basic ones. To the extent that states are defined as expectation-value functionals on the algebra of observables, their “nature” is exhausted in accomplishing the task of computing the expectation values of the observables of the algebra. In other words, states are not to be confused with observables, they are not to be understood as any kind of property of the quantum system. As Earman (2015: 324) emphasizes, one should never forget “the mantra of the algebraic approach: a system state is an expectation value functional on the system algebra.”

4.4.2 The Structure of the Ontology

Following the Wittgensteinian idea that the structure of language is also the structure of reality, the latter arises by establishing the ontological counterpart of the algebraic formalism, that is, by providing an interpretation for each physical/mathematical term.

- The term ‘observable’ is used in quantum physics to refer to certain quantifiable magnitudes of physical relevance, which are mathematically represented by Hermitian operators. Ontologically, they correspond to items belonging to the category of property, in particular, determinable properties, which here will be referred to as ‘type-properties’. In addition, it is necessary to distinguish between *universal type-properties (U-type-properties)* and *instances of universal type-properties (I-type-properties)*. The ontological counterparts of general physical magnitudes are *U-type-properties*, and of observables are *I-type-properties*. We will symbolize an U-type-property as $[A]$, and its I-type-properties as $[A^i]$. An example of U-type-property is energy $[H]$, which can be instantiated as the energy $[H^1]$ of *this* particular system. Let us stress that, although this talk suggests an ontology of objects, below we will define the concept of quantum system as a non-objectual ontological item.
- Since a physical observable is a quantifiable magnitude, it has different possible values, which are mathematically represented by the eigenvalues of the corresponding Hermitian operator. Their ontological counterparts are determinate properties, which here will be referred to as ‘*possible case-properties*’ (*P-case-properties*) of the corresponding I-type-property. Here the terms ‘type-properties’ and ‘case-properties’ stand for determinables and determinates, respectively; they are used just to emphasize that the corresponding items belong to the ontological category of property. Given an I-type-property $[A^i]$ of a U-type-property $[A]$, its P-case-properties will be symbolized as $[a_j^i]$. Following with

the above example, we can talk of the P-case-properties $[\omega_j^1]$ (the energy values that constitute the energy spectrum) of the energy $[H^1]$ of this particular system, where $[H^1]$ is an I-type-property of the U-type-property energy $[H]$.

- In physics it is implicitly assumed that each observable, although having multiple possible values, cannot have more than one value at a time. The value actually acquired by an observable has no direct mathematical representation in the theory: there is no formal way to distinguish it from the remaining possible values. But, ontologically, it is essential to emphasize that, given an I-type-property $[A^i]$ of a U-type-property $[A]$, *no more than one* of its P-case-properties $[a_j^i]$ becomes *actual*. That *actual case-property* (*A-case property*) will be symbolized as $[a_k^i]$. In the above example, $[\omega_k^1]$ is the actual value of the energy $[H^1]$ of this particular system. Notice that the clause “no more than one”, which corresponds to “exactly one” in the classical case, can be “zero” in the quantum case. In fact, as the Kochen–Specker theorem shows, not all the I-type-properties of a system have an *A-case property*.
- Since in the algebraic formalism the physical concept of quantum system S^i is mathematically represented by an algebra of observables, its ontological counterpart is a *bundle* $\mathfrak{B}^i = \{[A^i], [B^i], [C^i], \dots\}$ of the I-type-properties $[A^i], [B^i], [C^i], \dots$ corresponding to the U-type-properties $[A], [B], [C], \dots$. The precise nature of these bundle-systems will be discussed in the following subsection.
- The physical concept of state is mathematically represented by an expectation-value functional over the space of observables. As mentioned above, in this interpretive framework states do not refer to properties but are endowed with a probabilistic nature. More precisely, the state of a system S^i encodes the ontological *propensities to actualization* of all the P-case-properties of all the I-type-properties belonging to the bundle $\mathfrak{B}^i$, which is the ontological counterpart of S^i .

In this ontological proposal, type-properties are conceived as universals. A legitimate question is why not to appeal to tropes. The answer is strongly linked to the problem of indistinguishability. Although they may be absolutely similar, tropes are neither absolutely indistinguishable nor only numerically different, precisely because they can be individuated and distinguished by their space-time position (redness here and now), by the object to which they apply (red of this individual balloon), or because the distinction between them is taken to be primitive (see Sect. 4.2.4). Since the elemental items of the quantum ontology should be adequate to provide the foundations of quantum indistinguishability, an ontology of tropes would face the same difficulties as an ontology of objects, since in both cases they are distinguishable items. By contrast, the instances of a universal are absolutely indistinguishable because they are manifestations of a same property: the roundness of a billiard ball and the roundness of a water drop are both instances of the universal roundness, and trying to distinguish them as different properties makes no sense. For this reason, an ontological approach based on universals and their

instances paves the way towards an ontologically reasonable answer to the problem of indistinguishability.

In Sect. 4.2.5, the importance of not confusing between the universal-instance relation and the determinable-determinate relation was emphasized by means of an example coming from classical mechanics: the position and the momentum of a particular billiard ball as determinable instances of the universal properties “position” and “momentum”, respectively, having determinate values with respect to a billiard table. The difference between the two relations is completely analogous in the quantum framework. The difference between the classical case and the quantum case lies not in those relationships, but in the role played by possibility and actualization. In the classical case, at a given time, a single possible determinate corresponds to each determinable and, as a consequence, such a determinate becomes actual. For example, at time t_1 , the determinable Q = “position of the billiard ball” has a single possible determinate, say, q_1 = “10 cm from corner of the table”, and this is the actual determinate position of the billiard ball at t_1 . In the quantum case, by contrast, at a given time, determinables may have many different possible determinates, among which at most only one becomes actual. For example, at a given time, the determinable S_z = “spin in direction z of the quantum system S ” has two possible determinates, S_{z1} = “spin up in direction z ” and S_{z2} = “spin down in direction z ”: it may be the case that one of them becomes actual; but, according to the Kochen–Specker theorem, it may also be the case that neither of them becomes actual.

Finally, it is worth introducing some remarks about the concept of possibility, whose nature has been one of the most controversial issues in the history of philosophy. Two general ways of conceiving possibility can be distinguished (see Menzel, 2022). According to *actualism*, everything that exists, when analyzed in depth, turns out to be actual: the discourse on possibility can be reduced to a language that only refers to what actually exists; as a consequence, the predicate ‘actual’ is redundant. For *possibilism*, by contrast, possibility is an ontologically irreducible feature of reality: possible items need not become actual in order to be real. In Aristotelian terms, being can be said in different ways: as possible being or as actual being. Given the essential probabilistic nature of quantum phenomena, in the present proposal possibility is conceived in non-actualist terms. An I-type-property has—possible—P-case-properties, among which at most one becomes actual, and the state gives the measure of the corresponding possibilities, that is, the measure of the tendency to actualization of those P-case-properties. These facts have nothing to do with a limitation of our knowledge about an underlying actual state of affairs. Probabilities measure possibilities conceived as propensities to actualization, which are ontologically irreducible because the theory is irreducibly indeterministic (see Lombardi et al., 2022).

4.4.3 *Non-objectual Bundles*

The problem of the nature of objects remains one of the main areas of controversy in contemporary metaphysics: is an object a substratum supporting properties or a mere “bundle” of properties? (for a survey, see Loux, 1998). The conception of an object as a substratum acting as a carrier of properties has pervaded the history of philosophy. As already mentioned, it is present under different forms, for example, in Aristotle’s “primary substance” and in Locke’s “substance in general”. However, following Hume’s rejection of the idea of substance, many philosophers belonging to the empiricist tradition, such as Bertrand Russell, Alfred Ayer, and Nelson Goodman, have regarded the postulation of a characterless substratum as a metaphysical abuse, and have adopted some version of the bundle theory. According to this view, an object is nothing but a bundle of properties: properties have metaphysical priority over objects and are therefore the fundamental items of the ontology.

In the literature, it has been argued that the difference between the substratum theory and the bundle theory is only verbal: in the bundle theory the object results from a “compresence relation” that serves the same purposes as the substance in the traditional substratum-plus-attributes theory. Therefore, the decision as to whether an object is a substratum supporting properties or simply a bundle of properties remains a matter of metaphysical taste (see Benovsky, 2008). While this may be the case in the classical domain, quantum mechanics calls this conclusion into question. Indeed, although in the present proposal quantum systems are ontologically characterized as bundles of properties, it is important to emphasize the peculiarity of this perspective.

The first point to consider is related to the difference between determinables and determinates, which is rarely taken into account in discussions about the ontological interpretation of quantum mechanics (for exceptions, see Calosi & Wilson, 2019; Calosi & Mariani, 2021). According to the traditional versions of the bundle theory, an object is the convergence of certain determinate properties, under the assumption that the determinable properties are all determinate. For example, a billiard ball is the confluence of a definite value of position, say here, a definite shape, say round, a definite color, say white, etc. So, the problem is to decide whether this object is a substratum in which definite position, roundness and whiteness inhere, or is the mere bundle of those determinate properties. But in both cases the properties that compose the bundle are *actual* properties. In the quantum case, by contrast, not all the determinable properties of a system are determinate; as a consequence, the system cannot be identified with a bundle of determinate properties. For this reason, in the present proposal, a quantum system is conceived as a bundle of determinables, that is, type-properties (I-type-properties), each one of them with its *possible* case-properties (P-case-properties). This is the first reason why this interpretation of the nature of quantum systems cannot be assimilated to the traditional notion of object.

On the other hand, in its traditional versions, the bundle theory is a theory about particular objects, according to which objects are composed of items of

a different ontological category (namely, properties). In other words, the bundle theory is designed to account for objects without appealing to a substratum on which properties inhere (see, e.g., O’Leary-Hawthorne, 1995; French, 2019). To this end, some properties must be selected to play the role of the principle that supplies synchronic and diachronic identity. The proposed quantum bundle view, by contrast, completely dispenses with the ontological category of object: bundles of properties do not behave as objects at all since they belong to a different ontological category. On this basis, when two bundle-systems combine, the composite system is also a bundle. And since bundles are not objects, there is no principle that preserves their identity in the composition: in the composite system the identity of the components is not preserved precisely because they are not objects at all. Also in this sense quantum systems are conceived as *non-objectual bundles* of properties.

Precisely because of their non-objectual nature, bundle-systems require a different kind of logic. An ontological picture in which properties are the elementary items, and do not constitute objects, is not adequately captured by any formal theory whose elementary symbols are individual variables referring to classical objects. But, as remarked in Sect. 4.2.2, most systems of logic are designed to handle individual objects. A way out of this problem is to develop a “logics of predicates” in the spirit of the “calculus of relations” proposed by Tarski (1941), in which individual constants and variables are absent. A different strategy is to apply quasi-set theory (see, e.g., Krause, 1992; da Costa and Krause, 1999): although it was originally devised to provide a formalism for indiscernible quantum objects, it can be adapted to formally deal with aggregates of items that do not belong to the ontological category of object but to that of property, so that bundles turn out to be represented by quasi-sets of properties (Holik et al., 2022).

4.5 Revisiting the Ontological Challenges

Let us insist again that metaphysics is underdetermined by physics; in particular, quantum mechanics is compatible with different ontological pictures. Thus, arguing in favor of a certain quantum ontology over others requires showing how fruitful it is in the task of offering reasonable solutions to interpretive problems. This section is devoted to show the advantages of the proposed picture for dealing with the ontological challenges of quantum mechanics: contextuality, non-locality, and indistinguishability.

4.5.1 Contextuality

As explained in Sect. 4.3.1, the Kochen–Specker theorem proves the impossibility of ascribing precise values to *all* the observables of a quantum system simultaneously, while preserving the functional relations between commuting observables. For

this reason, the contextuality resulting from the theorem defies the principle of omnimode determination, according to which all determinables are determinate in any object—individual or stuff. It is in this sense that quantum systems cannot be conceived of as classical objects: in any situation, they have determinable properties that are not determinate.

In our ontological language, quantum contextuality is expressed by saying that, given a bundle, not all of its I-type-properties actualize, that is, acquire an (actual) A-case-property among all their (possible) P-case-properties: which I-type-properties acquire an A-case-property is a “contextual” fact. Of course, in each context one could insist on the classical idea of I-type-properties with their definite A-case-properties with no contradiction. In other words, the picture of a bundle of actual case-properties that stands for a classical object could be retained in each context. But as soon as we try to extend this ontological picture to all the contexts by conceiving the object as a bundle of bundles, the Kochen–Specker theorem imposes an insurmountable barrier: in a quantum system, it is not possible to actually ascribe A-case-properties corresponding to all the I-type-properties in a non-contradictory manner. Therefore, the classical idea of a bundle of bundles of *actual* case-properties does not work in the quantum ontology.

The Kochen–Specker theorem introduces a constraint with respect to A-case-properties, more precisely, with respect to which P-case-properties of a bundle can enter actuality. But this restriction does not affect our concept of quantum system, because it is defined not as a bundle of actual case-properties, as in the traditional bundle theory, but as a bundle of I-type-properties, each with its corresponding *possible* P-case-properties. Precisely for this reason these quantum bundles do not constitute objects in the traditional sense: they are non-objectual bundles. Since the ontology is only populated by properties and bundles of properties, the principle of omnimode determination, valid for objects, is not false but does not apply. As a consequence, this ontology, devoid of objects, is immune to the challenge represented by the Kochen–Specker theorem.

4.5.2 *Non-locality and Non-separability*

As recalled in Sect. 4.3.2, the quantum domain seems to admit correlations between the properties of distant non-interacting systems, strongly suggesting non-local influences between distant systems that are incompatible with special relativity. Despite disagreements about this particular feature of quantum mechanics, in general the arguments about non-locality are based on the assumption that quantum systems are individual objects, and subsystems are also individuals. Consequently, the problem is to explain how the properties of those individual subsystems are instantaneously correlated even though they are not in interaction and they are located in different spatial positions. However, from an ontology of properties, the problem appears in a new light.

Recall that the ontological category of individual requires some “principle of individuality” that, regardless of its specific form, identifies a particular individual as distinct from another and as the same over time (see discussion in French & Krause, 2006). In turn, individuals can form aggregates, in which they can be counted (see Sect. 4.2.2): “countability” depends on the possibility of ontologically distinguishing each individual from the others. Therefore, if two individual systems interact as to yield a composite system in an entangled state, they should retain their identity as individual parts of the new whole. For this reason, EPR-correlations are conceived as the correlations linking the properties of those individual subsystems, and when they are distant in space and do not interact, such correlations become puzzling.

By contrast, in our ontology of properties, quantum systems are not individuals, not even objects: they are non-objectual bundles of properties. Therefore, there is no principle of individuality that allows them to retain their individual identity when they merge into a new bundle-system. As a consequence, the issue of interpreting EPR-correlations acquires a new formulation from the outset. The problem is no longer to explain the correlations between the properties of distant non-interacting objects. Since the composite bundle is a single whole, non analyzable in component bundles, the EPR-correlations are correlations between properties of a single item. Thus, the mystery of the original formulation, which seems to require a certain unexplainable harmony between distant objects, vanishes, since correlations between properties of a single system are natural even in the classical ontological domain. For example, it is not surprising that the area of a table is correlated to its length, or that the kinetic energy of a car is correlated to its mass and to its velocity: there is no need of an enigmatic harmony to explain these correlations.

In a sense, this view implies a kind of holism. However, in a traditional ontology of objects and properties, the indivisible whole is also an individual. But, according to the traditional view, an individual, if it is not “atomic”, can be split up giving rise to individual parts that are different from the original one (see Sect. 4.2.2). This implies that, from a holistic perspective framed in a traditional ontology, the challenge is to account for the fact that the individual composite system is a whole that cannot be decomposed into individual parts. This problem also disappears in an ontology that lacks the category of individual: since the holistic item is not an individual, the fact that it lacks individual parts turns out to be an expected consequence.

An ontology of properties without objects also allows us to cope with the fact that any composite quantum system can be decomposed into subsystems in different ways, none of them privileged over the others. This fact makes entanglement essentially relative to the particular decomposition considered in each case. Therefore, if subsystems are conceived as individuals, it must be accepted that there may be multiple non-local entanglement-induced correlations between multiply defined individual subsystems, which must be accounted for. From the perspective of our ontology, by contrast, a quantum system is a single non-objectual bundle of properties. Thus, the relativity of entanglement with respect to the multiple partitions of the composite system is nothing but the manifestation of the multiple

correlations between the properties of a single bundle, which have nothing to do with non-local influences between non-univocally defined individual subsystems.

This way of conceiving quantum correlations can be expressed mathematically in the algebraic formalism of quantum mechanics. Although the relativity of entanglement is usually introduced in terms of different tensor product structures—the various ways in which a Hilbert space can be decomposed into a tensor product of Hilbert spaces—, some authors have diverted their attention from Hilbert spaces to algebras of observables. For instance, Paolo Zanardi and his collaborators have taken an algebra $\mathcal{M}$ on a finite Hilbert space $\mathcal{H}$ as the starting point, to prove that, given two independent subalgebras $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{M}$ that satisfy (i) independence ($[\mathcal{A}, \mathcal{B}] = 0$, that is, $[a, b] = 0$ for all $a \in \mathcal{A}$ and $b \in \mathcal{B}$) and (ii) completeness ($\mathcal{A} \otimes \mathcal{B} \cong \mathcal{A} \vee \mathcal{B} = \mathcal{M}$), then $\mathcal{A}$ and $\mathcal{B}$ induce a tensor product $\mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$ (Zanardi, 2001; Zanardi et al., 2004). The authors stress that, in this way, the partition of the algebra of observables and the resulting entanglement of the state of interest can be made to depend on the observables accessible in each situation. In turn, Nathan Harshman and Kedar Ranade (2011) provide an explicit constructive method for generating such subalgebras from a finite set of operators that, although may look arbitrary from the viewpoint of the unstructured Hilbert space, have the right properties to rigorously define locality, separability, and entanglement. This algebraic perspective on entanglement also dispenses with the concept of particle—individual—and places observables—properties—at the center of the scene.

4.5.3 Indistinguishability

As already explained, the problem of indistinguishability arises in quantum statistics when the issue is to explain why a permutation of individual particles does not lead to a different complexion. Consequently, particles are considered indistinguishable, leading to the violation of the weakest version of the principle of identity of indiscernibles. As pointed out in Sect. 4.3.3, despite the Received View about indistinguishability and its critics differ with respect to conceiving quantum systems as individuals, they nevertheless agree with respect to subsuming them under the category of object. In this subsection it will be argued that, by dispensing with the category of object, the problem acquires a completely different formulation.

According to the Received View, quantum systems are non-individuals; it has also been suggested that they are not even objects at all (Quine, 1976, 1990). But these views give no metaphysical characterization of those items beyond their non-individuality or non-objectuality: they are only negatively characterized. By contrast, in the proposed ontology of properties, non-objectual quantum systems are positively and precisely characterized in metaphysical terms as bundles of I-type-properties. Moreover, I-type-properties, with their corresponding P-case-properties, are ontological items metaphysically characterized in a clear way and physically/formally represented with precision by observables of an algebra. As

will be argued below, this positive characterization makes it possible to draw many conclusions about the nature and the behavior of quantum systems.

In the traditional treatment of the problem, indistinguishability is a relation between particles, that is, individuals: particles with the same state-independent properties are indistinguishable. By contrast, in an ontology of properties, indistinguishability is primarily a relation between two instances of a same universal type-property when they have the same case-properties: two I-type-properties $[A^1]$ and $[A^2]$ are indistinguishable when they are I-type-properties of the same U-type-property $[A]$ and they have the same P-case-properties, $[a_j^1] = [a_j^2]$. From this primary meaning, indistinguishability acquires a derived meaning when applied to bundles: two bundle-systems are indistinguishable when their respective I-type-properties are indistinguishable. Both indistinguishable I-type-properties and indistinguishable bundle-systems are only numerically different. Nevertheless, this does not imply that the principle of identity of indiscernibles is false for them: whereas the principle refers to the identity of indiscernible objects, in our case indistinguishability is a relation between items belonging to the ontological category of property. It is precisely this positive characterization of quantum systems that makes the non-applicability of the principle to them conceptually meaningful.

When indistinguishable bundles combine, it is natural to expect that the I-type-properties belonging to the new bundle do not distinguish between the original bundles. Simply phrased, when two indistinguishable bundles merge into a single whole, which component bundle is taken first and which second does not matter at all. Mathematically, this requires that the observables representing the I-type-properties belonging to the composite bundle-system be symmetric with respect to the permutation of the component bundles. In this way, the so-called “indistinguishability principle” is satisfied in a natural way. In fact, in the context of the traditional particle-view, the principle states that all quantum states that differ only by a permutation of indistinguishable particles are observationally indistinguishable, that is, they lead to the same expectation values for any observable of the system. This requirement can be satisfied by restricting states to be symmetric (bosonic) or anti-symmetric (fermionic), or by restricting observables to be symmetric (see Messiah & Greenberg, 1964). Both the assumptions that certain states are inaccessible and that certain observables are not allowed have a certain ad hoc flavor, since they are posited exclusively to satisfy the indistinguishability principle. By contrast, in the proposed ontology of properties, the restriction on observables is ontologically motivated. The observables of systems composed of indistinguishable subsystems are symmetric due to the very nature of the properties of the component bundle: they are indistinguishable and, then, the order in which they are incorporated into the composite bundle is absolutely irrelevant. Therefore, the indistinguishability principle need not be considered an ad hoc postulate of the theory, but turns out to be a consequence of the ontologically motivated symmetry of the observables of the composite system. As an additional advantage, dispensing with symmetrization and anti-symmetrization of states dissolves the traditional problems of defining entanglement in the case of indistinguishability (see Fortin & Lombardi, 2022).

4.6 The Physical Nature of Non-objectual Bundles

4.6.1 Which Properties?

Up to this point, the quantum ontology is described as a property-only realm, in which systems are non-objectual bundles of properties. However, this characterization remains formal, as it says nothing about which properties effectively inhabit the quantum domain. In fact, not any I-type-property can belong to a quantum bundle-system. The properties involved in the quantum ontology are *physical* properties.

As Leslie Ballentine (1998) points out, although the formal structure of quantum mechanics is a necessary basis for the formulation of the theory, it has by itself very little physical content. When concrete physical problems are to be solved, the relevant observables of the system, endowed with a clear physical meaning, must be identified. Those observables are closely related to space-time symmetry transformations.

Let us begin by recalling that each physical theory has a corresponding group of symmetry transformations, in the sense that the dynamical law of the theory is covariant under the transformations of the group, that is, it preserves its form under these transformations. The group corresponding to quantum mechanics is the Galilei group. Since it is a Lie group, each Galilei transformation T_α can be represented by a unitary operator U_α , with the exponential parametrization $U_\alpha = e^{iK_\alpha s_\alpha}$, where s_α is a continuous parameter and K_α is a Hermitian operator independent of s_α , called “generator” of the transformation T_α . So, the Galilei group is defined by ten symmetry generators associated to ten parameters: one time-displacement, three space-displacements, three space-rotations, and three boost-velocity components. Those symmetry generators represent the basic physical observables of the theory (strictly speaking, the generators are proportional to the corresponding observables with a factor $1/\hbar$): the energy H (time-displacement), the momentum $P = (P_x, P_y, P_z)$ (space-displacement), the total angular momentum $J = (J_x, J_y, J_z)$ (space-rotation), and the position $Q = (Q_x, Q_y, Q_z)$ (boost-transformation, whose generator is mQ , where m is the mass).

It is worth noting that, if the Hamiltonian H is a function of time, in general it cannot be conceived of as the generator of time-displacements. This means that the time-independence of the Hamiltonian is what endows the Schrödinger equation with a clear physical meaning (precisely, that of expressing time-displacements) and, at the same time, what makes it strictly applicable to closed systems. This result implicitly supports the orthodox formulation of quantum mechanics, in which the quantum system is conceived as a closed, constant-energy system, which unitarily evolves according to the Schrödinger equation. The Hamiltonian (the energy) of the system only changes with time as the result of its interaction with other systems.

Although in a—closed—quantum system the Hamiltonian H is time-independent and, then, invariant under time-displacements, it may or may not have the remaining space-time symmetries. When H is invariant under a certain continuous transformation, the generator of that transformation is a *constant of motion* of the system. In

other words, each symmetry of the Hamiltonian defines a conserved quantity. For example, the invariance of H under space-displacements in any direction implies that the momentum P is a constant of motion; the invariance of H under space-rotations in any direction implies that the total angular momentum J is a constant of motion. If, on the contrary, H is invariant under space-displacements only in one direction, say x , only the component P_x of P is a constant of motion. The central role played by the Hamiltonian in the dynamical law of the theory and in the definition of the constants of motion of the system has led some authors to consider it the touchstone of the interpretation of quantum mechanics. According to the Modal-Hamiltonian Interpretation (Lombardi & Castagnino, 2008; Ardenghi et al., 2009a, b; Lombardi et al., 2010; Fortin et al., 2018; Lombardi & Ardenghi, 2022), the Hamiltonian of the quantum system defines the preferred context, that is, the set of the observables—I-type-properties—that acquire an actual definite value—an A-case-property—among their possible values—P-case-properties.

In summary, space-time symmetry transformations endow the formal skeleton of quantum mechanics with the physical flesh and blood that make it a well-specified physical theory. From the ontological viewpoint, they play a central role in the identification of the fundamental physical properties of the quantum realm.

4.6.2 What Holds Properties Together?

According to the traditional bundle theory, objects are composed of items of a different category: they are bundles of properties. However, not just any collection of properties forms a bundle that is an object: following Russell (1940), properties must hold a relation that binds them together in order to constitute an object. Objects are either bundles of coinstantiated universals in the universals-view or bundles of compresent tropes in the tropes-view. Both *coinstantiation* and *compresence*, which tie properties together, are commonly regarded as primitive relations, serving the same purposes as substance or bare particulars in the traditional object-and-properties ontological view.

In the quantum case, not every type-property of the bundle-system has an actual case-property. For this reason, in the present ontological proposal, a quantum system is not a bundle of actual case-properties but a bundle of I-type-properties, that is, instances of universal type-properties, and the bundle itself is formally represented by an algebra of observables. Bundling, in this case, does not require a coinstantiation or compresence relation that plays the role of the substance in equipping the bundle with a feature that distinguishes it from other bundles and reidentifies it over time. Such relations are not necessary precisely because bundles are not objects.

However, does this mean that quantum bundles are mere collections or aggregates of properties, with nothing holding them together? The answer to this question is negative. Bundle-systems have a well-defined structure, given by the specific relations that link the I-type-properties of the bundle to each other. Those relations

are formally represented by the *commutation relations* of the form $[A, B] = C$ between the observables of the algebra. In other words, what “ties up” the I-type-properties of a bundle is the physically meaningful structure of relations accurately represented by the mathematical structure of the corresponding algebra of observables.

In turn, that structure of relations is neither vaguely defined nor arbitrary, but rather it also follows from the symmetry group of quantum mechanics. As a Lie group, the Galilei group is defined by the commutation relations between its generators, which, as explained in the previous subsection, represent the basic physical observables of the theory:

(a) $[P_i, P_j] = 0$	(f) $[G_i, P_j] = i\delta_{ij}M$
(b) $[G_i, G_j] = 0$	(g) $[P_i, H] = 0$
(c) $[J_i, J_j] = i\epsilon_{ijk}J_k$	(h) $[J_i, H] = 0$
(d) $[J_i, P_j] = i\epsilon_{ijk}P_k$	(i) $[G_i, H] = iP_i$
(e) $[J_i, G_j] = i\epsilon_{ijk}G_k$	

where $\hbar$ is taken as equal to one, and ϵ_{ijk} is the Levi-Civita tensor, such that $i \neq k$, $j \neq k$, $\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij} = 1$, $\epsilon_{ikj} = \epsilon_{jik} = \epsilon_{kji} = -1$, and $\epsilon_{ijk} = 0$ if $i = j$. The rest of the physical magnitudes can be defined in terms of these basic ones: for instance, the three position components are $Q_i = G_i/m$, the three orbital angular momentum components are $L_i = \epsilon_{ijk}Q_jP_k$, and the three spin components are $S_i = J_i - L_i$. In turn, the Galilei group has three Casimir operators which, as such, commute with all the generators of the group: they are the mass operator M , the spin-squared operator S^2 , and the internal energy operator $W = H - P^2/2m$. The eigenvalues of the Casimir operators label the irreducible representations of the group; so, in each irreducible representation, the Casimir operators are multiples of the identity: $M = mI$, where m is the mass, $S^2 = s(s+1)I$, where s is the eigenvalue of the spin S , and $W = wI$, where w is the scalar internal energy.

In his seminal paper, Eugene Wigner (1939) introduced the idea that kinds of elementary particles in a quantum theory are represented by the irreducible projective representations of the symmetry group of the underlying space-time corresponding to that theory. In that paper, he focused on quantum field theory in Minkowski space-time, claiming that elementary particles correspond to the irreducible projective representations of the Poincaré group, which is the symmetry group of the Minkowski space-time. But this idea can also be applied to non-relativistic quantum mechanics, in such a way that, in this theoretical framework, each irreducible representation of the Galilei group represents a kind of elemental particle, characterized by its mass m , its spin s , and its internal energy w (see da Costa et al., 2013).

This section has shown how the symmetry group of quantum mechanics endows the structure of the quantum ontology with a precise physical referent. In this sense, the Galilei group plays a dual role. On the one hand, it defines the physical content of the properties that make up the bundle-systems. On the other hand, it establishes the relations between these properties, which give cohesion to the bundle without

turning it into an object of traditional metaphysics. From a more general point of view, this section shows that physics and metaphysics are not at odds at all. On the contrary, when metaphysical notions are elucidated from the outset, they acquire physical content in a natural way.

4.7 Final Remarks

The interpretation of quantum mechanics has been discussed for over a hundred years because it challenges certain basic assumptions of traditional metaphysics. The usual strategy for dealing with this situation has been to focus on one of the various challenges posed by the theory and to devise an interpretation that solves it, leaving aside the remaining difficulties. However, one can aspire to formulate a “global” solution, according to which all the problems can be adequately addressed in terms of a single ontology. This was the aim of the present work.

Here it was argued that the ontological problems are not a matter of kinds of objects, but of ontological categories: it is necessary to decide how the reality referred to by quantum mechanics is structured, which ontological categories underlie the quantum realm. In the light of this central goal, the interpretive obstacles were addressed from a radical position: there are no individuals, not even objects in the quantum ontology; the quantum world is populated by quantum properties that form bundles which, nevertheless, do not acquire the necessary features to be subsumed under the ontological category of object. First, it was shown how this ontological picture provides coherent and conceptually unified answers to the main quantum ontological challenges: contextuality, non-separability and indistinguishability. Second, it was emphasized that the structure of this ontology can be endowed with precise physical content on the basis of the symmetry group of the theory.

Of course, what has been said in this chapter does not exhaust all the interpretative issues surrounding quantum mechanics. For example, one question that cannot be ignored concerns the nature of quantum possibility and, with it, the interpretation of probability. Another inescapable issue is that referred to how to talk about non-objectual quantum systems, given that our ordinary and formal languages are designed to describe an ontology of objects and properties. Connecting the two issues is the need for a logic that makes possible to speak not only of properties without objects, but particularly of possible properties, that is, a modal-property based logic. However, a detailed treatment of these matters is beyond the scope of the present chapter and will be addressed in future works.

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References

- Ardenghi, J. S., Castagnino, M., & Lombardi, O. (2009a). Quantum mechanics: Modal interpretation and Galilean transformations. *Foundations of Physics*, 39, 1023–1045.
- Ardenghi, J. S., Castagnino, M., & Lombardi, O. (2009b). Modal-Hamiltonian interpretation of quantum mechanics and Casimir operators: The road to quantum field theory. *International Journal of Theoretical Physics*, 50, 774–791.
- Armstrong, D. M. (1989). *Universals: An opinionated introduction*. Westview.
- Armstrong, D. M. (1993). A world of states of affairs. *Philosophical Perspectives*, 7, 429–440.
- Ballentine, L. (1998). *Quantum mechanics: A modern development*. World Scientific.
- Beltrametti, E., & Cassinelli, G. (1981). *The logic of quantum mechanics*. Addison-Wesley.
- Ben-Menahem, Y. (2018). *Causation in science*. Princeton University Press.
- Benovsky, J. (2008). The bundle theory and the substratum theory: Deadly enemies or twin brothers? *Philosophical Studies*, 141, 175–190.
- Berkovitz, J. (2016). Action at a distance in quantum mechanics. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Spring 2016 Edition). <https://plato.stanford.edu/archives/spr2016/entries/qm-action-distance/>
- Bernoulli, J. (1713). *Ars Conjectandi, Opus Posthumum. Accedit Tractatus de Seriebus Infinitis, et Epistola Gallice Scripta de Ludo Pilae Reticularis*. Thurneysen.
- Black, M. (1952). The identity of indiscernibles. *Mind*, LXI, 153–164.
- Bohm, A., & Gadella, M. (1989). Dirac Kets, Gamow Vectors and Gel'fand Triplets. In *The rigged Hilbert space formulation of quantum mechanics* (Springer lecture notes in physics) (Vol. 348). Springer.
- Bub, J., & Clifton, R. (1996). A uniqueness theorem for interpretations of quantum mechanics. *Studies in History and Philosophy of Modern Physics*, 27, 181–219.
- Calosi, C., & Mariani, C. (2021). Quantum indeterminacy. *Philosophy Compass*, 2021, 1–15.
- Calosi, C., & Wilson, J. (2019). Quantum metaphysical indeterminacy. *Philosophical Studies*, 176, 2599–2627.
- Campbell, K. (1990). *Abstract Particulars*. Basil Blackwell.
- da Costa, N., & Krause, D. (1999). Set-theoretical models for quantum systems. In M. L. Dalla Chiara, R. Giuntini, & F. Laudisa (Eds.), *Language, Quantum, Music* (pp. 171–181). Kluwer.
- da Costa, N., & Lombardi, O. (2014). Quantum mechanics: Ontology without individuals. *Foundations of Physics*, 44, 1246–1257.
- da Costa, N., Lombardi, O., & Lastiri, M. (2013). A modal ontology of properties for quantum mechanics. *Synthese*, 190, 3671–3693.
- Davidson, D. (1967). Truth and meaning. *Synthese*, 17, 304–323.
- Dieks, D. (2005). Quantum mechanics: An intelligible description of objective reality? *Foundations of Physics*, 35, 399–415.
- Dürr, D., Goldstein, S., & Zanghì, N. (2013). *Quantum physics without quantum philosophy*. Springer.
- Earman, J. (2015). Some puzzles and unresolved issues about quantum entanglement. *Erkenntnis*, 80, 303–337.
- Esfeld, M. (2019). Individuality and the account of nonlocality: The case for the particle ontology in quantum physics. In O. Lombardi, S. Fortin, C. López, & F. Holik (Eds.), *Quantum worlds: Perspectives on the ontology of quantum mechanics* (pp. 222–244). Cambridge University Press.
- Fortin, S., & Lombardi, O. (2022). Entanglement and indistinguishability in a quantum ontology of properties. *Studies in History and Philosophy of Science*, 91, 234–243.
- Fortin, S., Lombardi, O., & Martínez González, J. C. (2018). A new application of the modal-Hamiltonian interpretation of quantum mechanics: The problem of optical isomerism. *Studies in History and Philosophy of Modern Physics*, 62, 123–135.
- French, S. (1989). Identity and individuality in classical and quantum physics. *Australasian Journal of Philosophy*, 67, 432–446.

- French, S. (2019). Identity and individuality in quantum theory. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2019 Edition). <https://plato.stanford.edu/archives/win2019/entries/qt-idind/>
- French, S., & Krause, D. (2006). *Identity in physics a historical, philosophical, and formal analysis*. Clarendon Press.
- Friebe, C. (2014). Individuality, distinguishability, and (non-)entanglement: A defense of Leibniz's principle. *Studies in History and Philosophy of Modern Physics*, 48, 89–98.
- Gallois, A. (2016). Identity over time. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2016 Edition). <https://plato.stanford.edu/archives/win2016/entries/identity-time/>
- Gelfand, I., & Naimark, M. (1943). On the imbedding of normed rings into the ring of operators in Hilbert space. *Matematicheskii Sbornik*, 54, 197–217.
- Ghirardi, G. C., Rimini, A., & Weber, T. (1986). Unified dynamics for microscopic and macroscopic systems. *Physical Review D*, 34, 470–491.
- Ghirardi, G. C., Grassi, R., & Benatti, F. (1995). Describing the macroscopic world: Closing the circle within the dynamical reduction program. *Foundations of Physics*, 25, 5–38.
- Haack, S. (1974). *Deviant logic*. Cambridge University Press.
- Haack, S. (1978). *Philosophy of logics*. Cambridge University Press.
- Harshman, N. L. (2012). Observables and entanglement in the two-body system. *AIP Conference Proceedings*, 1508, 386–390.
- Harshman, N. L., & Ranade, K. S. (2011). Observables can be tailored to change the entanglement of any pure state. *Physical Review A*, 84, 012303.
- Healey, R. & Gomes, H. (2022). Holism and nonseparability in physics. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/physics-holism/>
- Holik, F., Jorge, J. P., Krause, D., & Lombardi, O. (2022). Quasi-set theory for a quantum ontology of properties. *Synthese*, 200, #401.
- Iguri, S., & Castagnino, M. (1999). The formulation of quantum mechanics in terms of nuclear algebras. *International Journal of Theoretical Physics*, 38, 143–164.
- Jauch, J. M., & Piron, C. (1969). On the structure of quantal propositional systems. *Helvetica Physica Acta*, 42, 842–848.
- Kant, I. (1902–). *Gesammelte Schriften*. Berlin: Herausgegeben von der Preußischen Akademie der Wissenschaften (Bde. 1–22), der Deutschen Akademie der Wissenschaften zu Berlin (Bd. 23), und der Akademie der Wissenschaften zu Göttingen (Bde. 24, 25, 27–29).
- Kaplan, D. (1975). How to Russell a Frege-Church. *The Journal of Philosophy*, 72, 716–729.
- Keinänen, M., & Hakkarinen, J. (2014). The problem of trope individuation: A reply to Lowe. *Erkenntnis*, 79, 65–79.
- Kochen, S., & Specker, E. (1967). The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, 17, 59–87.
- Krause, D. (1992). On a quasi-set theory. *Notre Dame Journal of Formal Logic*, 33, 402–411.
- Kuhlmann, M. (2010). *The ultimate constituents of the material world – In search of an ontology for fundamental physics*. Ontos-Verlag.
- Laycock, H. (2010). Object. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Fall 2010 Edition). <http://plato.stanford.edu/archives/fall2010/entries/object/>
- Lewowicz, L. (2005). *Del Relativismo Lingüístico al Relativismo Ontológico en el Último Kuhn*. Departamento de Publicaciones de la Facultad de Humanidades y Ciencias de la Educación, Universidad de la República.
- Lewowicz, L., & Lombardi, O. (2013). Stuff versus individuals. *Foundations of Chemistry*, 15, 65–77.
- Lombardi, O., & Ardenghi, J. S. (2022). How different interpretations of quantum mechanics can enrich each other: The case of the relational quantum mechanics and the modal-Hamiltonian interpretation. *Foundations of Physics*, 52, #64.
- Lombardi, O., & Castagnino, M. (2008). A modal-Hamiltonian interpretation of quantum mechanics. *Studies in History and Philosophy of Modern Physics*, 39, 380–443.

- Lombardi, O., & Dieks, D. (2016). Particles in a quantum ontology of properties. In T. Bigaj & C. Wüthrich (Eds.), *Metaphysics in contemporary physics* (pp. 123–143). Brill-Rodopi.
- Lombardi, O., Castagnino, M., & Ardenghi, J. S. (2010). The modal-Hamiltonian interpretation and the Galilean covariance of quantum mechanics. *Studies in History and Philosophy of Modern Physics*, 41, 93–103.
- Lombardi, O., Fortin, S., & Pasqualini, M. (2022). Possibility and time in quantum mechanics. *Entropy*, 24, #249.
- Loux, M. (1998). *Metaphysics. A contemporary introduction*. Routledge.
- MacLeod, M. & Rubenstein, E. (2006). Universals. In J. Fieser & B. Dowden (eds.), *The internet encyclopedia of philosophy*. <https://iep.utm.edu/universa/>
- Maurin, A.-S. (2018). Tropes. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Summer 2018 Edition). <https://plato.stanford.edu/archives/sum2018/entries/tropes/>
- McKay, T. (2008). Critical notice of *words without objects*. *Canadian Journal of Philosophy*, 38, 301–323.
- Menzel, C. (2022). The possibilism-actualism debate. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/possibilism-actualism/>
- Messiah, A. M. L., & Greenberg, O. W. (1964). Symmetrization postulate and its experimental foundation. *Physical Review B*, 136, 248–267.
- Muller, F. A. & Saunders, S. (2008). Discerning fermions. *The British Journal for the Philosophy of Science*, 59, 499–548.
- Noonan, H. & Curtis, B. (2022). Identity. In E. N. Zalta & U. Nodelman (Eds.), *Stanford encyclopedia of philosophy* (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/identity/>
- O’Leary-Hawthorne, J. (1995). The bundle theory of substance and the identity of indiscernibles. *Analysis*, 55, 191–196.
- Orilia, F. & Paolini Paoletti, M. (2022). Properties. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/properties/>
- Pelletier, F. J. (Ed.). (1979). *Mass terms: Some philosophical problems*. Reidel Publishing Company.
- Piron, C. (1976). *Foundations of quantum physics*. W. A. Benjamin.
- Post, H. (1963). Individuality and physics. *The Listener*, 70, 534–537.
- Quine, W. V. O. (1960). *Word and object*. MIT Press.
- Quine, W. V. O. (1976). Whither physical objects? In R. S. Cohen, P. Feyerabend, & M. Wartofsky (Eds.), *Essays in memory of Imre Lakatos* (pp. 497–504). Reidel.
- Quine, W. V. O. (1990). *The pursuit of truth*. Harvard University Press.
- Rettler, B. & Bailey, A. M. (2022). Object. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/object/>
- Russell, B. (1914). *Our knowledge of the external world*. Allen and Unwin.
- Russell, B. (1940). *An inquiry into meaning and truth*. Allen and Unwin.
- Saunders, S. (2003). Physics and Leibniz’s principles. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 289–307). Cambridge University Press.
- Schaffer, J. (2001). The individuation of tropes. *Australasian Journal of Philosophy*, 79, 247–259.
- Segal, I. E. (1947). Irreducible representations of operator algebras. *Bulletin of the American Mathematical Society*, 53, 73–88.
- Tarski, A. (1941). On the calculus of relations. *The Journal of Symbolic Logic*, 6, 73–89.
- Tegtmeier, E. (2000). Events as facts. In J. Faye, U. Scheffler, & M. Urchs (Eds.), *Things, facts, and events* (pp. 219–228). Rodopi.
- Teller, P. (1998). Quantum mechanics and haecceities. In E. Castellani (Ed.), *Interpreting bodies: Classical and quantum objects in modern physics* (pp. 114–141). Princeton University Press.
- Terra Cunha, M. O., Dunningham, J. A., & Vedral, V. (2007). Entanglement in single-particle systems. *Proceedings of the Royal Society A*, 463, 2277–2286.

- Thomason, S. K. (1989). Free construction of time from events. *Journal of Philosophical Logic*, 18, 43–67.
- Tugendhat, E. (1982). *Traditional and analytical philosophy: Lectures on the philosophy of language*. Cambridge University Press.
- Tumulka, R. (2006). A relativistic version of the Ghirardi-Rimini-Weber model. *Journal of Statistical Physics*, 125, 821–840.
- van Fraassen, B. C. (1985). Statistical behaviour of indistinguishable particles: Problems of interpretation. In P. Mittelstaedt & E.-W. Stachow (Eds.), *Recent developments in quantum logic* (pp. 161–187). Cologne.
- Whitehead, A. N. (1929). *Process and reality. An essay in cosmology*. Macmillan.
- Wigner, E. P. (1939). On unitary representations of the inhomogeneous Lorentz group. *Annals of Mathematics*, 40, 149–204.
- Wilson, N. L. (1974). Facts, events, and their identity conditions. *Philosophical Studies*, 25, 303–321.
- Wilson, J. (2022). Determinables and determinates. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy*, (Winter 2022 Edition). <https://plato.stanford.edu/archives/win2022/entries/determinate-determinables/>
- Wittgenstein, L. (1921). Logisch-Philosophische Abhandlung, *Annalen der Naturphilosophische*, XIV(3/4). English translation: C. K. Ogden (1922). *Tractatus Logico-Philosophicus*. Routledge & Kegan Paul.
- Wolff, Ch. (1728). *Philosophia Rationalis Sive Logica*. Reprint of the 1740 edition with introduction, notes and index in (1980) by Jean École (Ed.). Georg Olms.
- Zanardi, P. (2001). Virtual quantum systems. *Physical Review Letters*, 87, #077901.
- Zanardi, P., Lidar, D. A., & Lloyd, S. (2004). Quantum tensor product structures are observable induced. *Physical Review Letters*, 92, #06042.
- Zurek, W. H. (1982). Environment-induced superselection rules. *Physical Review D*, 26, 1862–1880.
- Zurek, W. H. (2003). Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics*, 75(715), 776.